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ABSTRACT

This document describes the nature of the Third Assessment of Mathematics for the National Assessment of Educational Progress. Information about the first two assessments is provided to give background information and details on the framework for the third assessment. It is noted that objectives for the third assessment are based on the second, with revisions that reflect current content and trends in school mathematics. Information about the content domain is provided, with the six content categories discussed: numbers and numeration; variables and relationships; shape, size, and position; measurement; probability and statistics; and technology. Technology is measured by assessing the use of calculators and computer literacy. Five categories in the process domain are reviewed: knowledge, skill, understanding, application and problem solving, and attitudes towards mathematics. Questions to be answered within each of the process categories are listed. The first of two appendices lists advisory committee members and consultants. The second provides additional information on three assessment topics: attitudes, the calculator, and estimation skills. (MP)

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MATHEMATICS OBJECTIVES

1981-82 Assessment

No. 13-MA-10

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CHAPTER 1

THE FIRST ASSESSMENT OF MATHEMATICS (1972-73)

The National Assessment of Educational Progress conducted the first assessment of mathematics during the 1972-73 school year. The objectives for that assessment were developed by two educational testing contractors: Educational Testing Service and Psychological Corporation. Each contractor independently developed a set of objectives, relying on its staff, mathematicians and mathematics educators. The final sets of objectives were reviewed by panels of interested lay citizens to decide which set to use in the assessment. Panel members were evenly divided in their preference for the two sets of objectives. In the absence of a strong preference, the Psychological Corporation was asked to continue the development of objectives, and in 1968, completed its revision of the objectives. The revision, together with objectives selected from the Educational Testing Service's version, was compiled into a final statement of objectives for the first assessment of mathematics. A booklet containing the statement was published in 1970.

When the objectives for mathematics were first formulated, they were compared with other statements of objectives that had appeared in mathematics education literature during the preceding 25 years. The objectives for the first assessment were consistent with objectives appearing in the literature. This outcome was both desired and expected since one criterion for the National Assessment objectives was that they be central to prevailing teaching efforts.

A three-dimensional classification scheme was used to categorize the mathematics objectives for the first assessment. One dimension of the scheme was "Uses of Mathematics," which was divided into three major categories:

1. Social mathematics (the mathematics needed for personal living and effective citizenship in our society).
2. Technical mathematics (the mathematics necessary for various skilled jobs and professions).
3. Academic mathematics (the formally structured mathematics that provide the basis for an understanding of various mathematical processes).

Another dimension of the matrix was "Content." The content areas were:

1. Numbers and numeration concepts.
2. Properties of numbers and operations.
3. Arithmetic computations.
4. Sets.
5. Estimation and measurement.
6. Exponents and logarithms.
7. Algebraic expressions.
8. Equations and inequalities.
9. Functions.
10. Probability and statistics.
11. Geometry.
12. Trigonometry.
13. Mathematical proof.
14. Logic.
15. Miscellaneous topics.
16. Business and consumer mathematics.
17. Attitude and interest.

The third dimension of the classification scheme consisted of six cognitive "Objectives or Abilities":

1. To recall and/or recognize definitions, facts and symbols.
2. To perform mathematical manipulations.
3. To understand mathematical concepts.
4. To solve mathematical problems — social, technical and academic.
5. To use mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions and verify results.
6. To appreciate and use mathematics.

During the development and review of the exercises, the content and ability dimensions of the classification scheme were the most useful. The exercise developers tended not to use the first dimension, uses of mathematics, when classifying exercises. This first dimension tended to pose too many restrictions on exercise development to make its use worthwhile.

Although the exercises were classified by content and ability, not all content areas or abilities were assessed equally. Certain content topics were purposely measured in more detail than others. Furthermore, even though the objectives were intended to include all the mathematics taught in the nation's schools, it was impossible to measure every objective in depth. Little emphasis, for example, was placed on the topics of trigonometry and logic. The content area of "attitude and interest" and the related ability of "appreciation and use

of mathematics" were not measured because the exercises developed to assess these were considered inadequate.

Approximately half of the exercises used in the mathematics assessment were released to the public. These exercises were included in various mathematics reports and made available to individuals, groups and states for their own uses. The unreleased exercises were used again in the second assessment of mathematics to measure changes in educational attainments. A general survey of the results of the first mathematics assessment is provided in *The First National Assessment of Mathematics: An Overview* (1975).[1] The text of each released exercise and accompanying documentation including results can be found in the *Mathematics Technical Report: Exercise Volume* (1977). Data are provided for all of the mathematics exercises, but the exact text and scoring guides are provided for the released exercises only.

Results concerning computational abilities of young Americans are presented and discussed in a special report, *Math Fundamentals: Selected Results From the First National Assessment of Mathematics* (1975). The results concerning consumer mathematics are presented and discussed in *Consumer Math: Selected Results From the First National Assessment of Mathematics* (1975). In addition, National Assessment has produced computer data tapes containing respondent-level data for the exercises that were reassessed in 1977-78. These data tapes are available to any researcher who wishes to conduct further analysis of the data.[2]

National Assessment has worked closely with a panel of mathematics educators from the National Council of Teachers of Mathematics (NCTM), who studied the data in order to draw implications from the results of the first mathematics assessment. The NCTM panel presented summaries of its findings in the October 1975 issues of *The Arithmetic Teacher* and *The Mathematics Teacher*. Additional brief articles on specific content topics were presented in the October 1975 through May 1976 issues of *The Arithmetic Teacher*. These articles suggest some of the ways mathematics teachers might use information from the first assessment to improve teaching. References for these articles are found in the section at the back of this booklet entitled "List of References."

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[1]National Assessment reports can be ordered through the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 or the National Assessment of Educational Progress, Suite 700, 1860 Lincoln Street, Denver, Colorado 80295.

[2]Data tapes are available, at a charge, through the Department of User Services, National Assessment of Educational Progress, Suite 700, 1860 Lincoln Street, Denver, Colorado 80295.

CHAPTER 2

THE SECOND ASSESSMENT OF MATHEMATICS (1977-78)

Unlike the first assessment, which made use of outside contractors, the objectives and exercises for the second assessment were developed through conferences organized by the National Assessment of Educational Progress. The conference procedure was intended to give the assessment greater flexibility, involve more professionals in mathematics and education and be more efficient in cost and time.

Several types of consultants participated in the developmental conferences: college or university mathematics educators, mathematicians, classroom teachers and interested lay citizens. The objectives had to be acceptable to these groups. The mathematics experts generally had to agree that the objectives were worthwhile and important to assess. The classroom teachers had to consider the objectives to be desirable teaching goals in most schools. Finally, the objectives had to be considered desirable by the lay citizens. The lay group, including parents and others with an interest in education, had to agree that an objective be important for America's youth to achieve and that it be of value in today's society.

The objectives were organized into a content-by-process matrix (see Figure 1). This matrix resembles the classification scheme developed for the first assessment and was used extensively in the developmental process. Fewer, but more inclusive, content and process headings in the matrix for the second assessment resulted in fewer cells. The complicated task of exercise development was subdivided into units corresponding to the cells of the matrix. The cells were weighted in proportion to their importance. The number of exercises to be assessed in each cell was determined by the relative weights, and review and selection were done by cells.

After the initial objectives matrix was developed, an advisory board was formed to give direction and advice to the National Assessment staff for further refinement of the objectives and the development of the assessment. The six-member advisory board included three university mathematics educators, two mathematicians and a mathematics teacher. This board was instrumental in organizing the final set of objectives, planning the development of exercises, selecting the final exercises and planning the subsequent reports.

FIGURE 1. Objectives Framework for the Second Assessment

PROCESS	CONTENT				
	A. Numbers and Numeration	B. Variables and Relationships	C. Shape, Size and Position	D. Measurement	E. Other Topics
I. Mathematical knowledge					
II. Mathematical skill					
III. Mathematical understanding					
IV. Mathematical application					

The first task of the advisory board was to review the new objectives matrix and put it into final form. The board and NAEP staff thought the objectives should be used as a plan or framework for exercise development and for reporting. They thought a reporting scheme should exist prior to exercise development to help organize and improve the comprehensiveness of the second assessment. The advisory board devised a set of questions that related to each of the four cognitive processes (knowledge, skill, understanding and application) described in Chapter 3. For example, under mathematical skills, one of the questions was "How well can students perform computation?" Under mathematical applications, one of the questions was "How well can students solve typical textbook problems?" Each series of questions was intended to be "answered" by the results from the assessment of a set of exercises. This planning helped insure that the questions could be adequately covered by the assessment. These questions and more detailed information on the objectives appear in *Mathematics Objectives, Second Assessment* (1978).

While the objectives were being formulated, conferences were held to discuss special topics reflecting current trends in mathematics education. A special topic that received considerable attention was the measurement of attitudes toward mathematics. Attitudes were not measured during the first mathematics assessment because of the difficulty of developing adequate exercises. However, consultants for the second assessment encouraged the development of attitudinal exercises, and an effort was made to develop such measures.

Assessment Results and Reports

Approximately one-third of the exercises from the second assessment were released, many of them appearing as examples in four reports on the results of the second assessment. The reports, reflecting the objectives matrix, are *Mathematical Knowledge and Skills* (1979), *Mathematical Understanding* (1979) and *Mathematical Applications* (1979). A fourth report, *Changes in Mathematical Achievement, 1973-78* (1979), discusses the changes in mathematical achievement during the five years between the first and second assessments.

The text of each released exercise and accompanying documentation including selected results can be found in *The Second Assessment of Mathematics, 1977-78, Released Exercise Set* (1979). Summary data augmenting the four selected reports mentioned above appear in *Mathematics Technical Report: Summary Volume* (1980). This report includes information on mean performance levels on various sets of items for the nation and various population subgroups. A detailed description of the developmental process, sampling, data collection, scoring and data analysis can be found in *Procedural Handbook: 1977-78 Mathematics Assessment* (1980). For researchers who are interested in doing further analyses of the assessment data, data tapes containing respondent-level data for all exercises in the second assessment are available.[1]

In addition to the reports produced by National Assessment, interpretive articles aimed primarily at teachers were written by a panel of mathematics educators from the National Council of Teachers of Mathematics (NCTM). The NCTM panel presented summaries of its findings in the April 1980 issue of *The Arithmetic Teacher* and the May 1980 issue of *The Mathematics Teacher*. Succeeding issues contain brief articles on specific topics, suggesting ways mathematics teachers might use the implications of the results in their teaching. Titles of these articles appear in the references listed at the back of this booklet.

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[1]Data tapes are available, at a charge, through the Department of User Services, National Assessment of Educational Progress, Suite 700, 1860 Lincoln Street, Denver, Colorado 80295.

CHAPTER 3

THE THIRD ASSESSMENT OF MATHEMATICS (1981-82)

The development of the objectives and assessment exercises proceeded along lines described in Chapter 2. College or university mathematics educators, mathematicians, classroom teachers and interested lay citizens contributed their views to the development of objectives and assessment exercises. A nine-member advisory committee was instrumental in reviewing the objectives and giving guidance throughout the developmental process. A list of the advisory board members and consultants who aided the developmental process appears in Appendix A.

The objectives for the third mathematics assessment are based on the framework used for the second assessment, with some revisions that reflect current content and trends in school mathematics. To update the objectives, numerous mathematicians and mathematics educators reviewed the second assessment objectives and made suggestions for revisions and new content.

The objectives framework uses a two-dimensional content-by-process matrix for organizing the objectives (see Figure 2). One dimension includes five process levels and the other dimension includes six content areas. Attitudes toward mathematics, ability to use the calculator and computer literacy (which were considered "special topics" in the last assessment) have been incorporated into the matrix for the third assessment.

Content

The content domain for the third assessment of mathematics draws primarily from the current curriculum of elementary and secondary schools, although some projection of future mathematics emphases is acknowledged (for example, assessment of problem solving strategies, use of calculators and computers). Mathematics up to, but not including, calculus is included in the assessment exercises, which are classified according to the six content categories shown in Figure 2.

These content categories help to organize the domain, but are not intended to be represented equally in the assessment. Each is discussed below.

FIGURE 2. Objectives Framework for the Third Assessment

PROCESS	CONTENT					
	A. Numbers and Numeration	B. Variables and Relationships	C. Shape, Size and Position	D. Measurement	E. Statistics/ Probability	F. Technology
I. Mathematical knowledge						
II. Mathematical skill						
III. Mathematical understanding						
IV. Mathematical application and problem solving						
V. Attitudes toward mathematics						

Numbers and Numeration

This category contains the largest number of exercises because of its importance in the curriculum. Exercises deal with the ways numbers are used, processed or written. Knowledge and understanding of numeration and number concepts are assessed for whole numbers, common fractions, decimal fractions, integers and percents. Considerable emphasis is placed on operations. Number properties and order relations are also included. Most of the exercises included here are to be done with paper and pencil; however, in some instances, students are asked to use other computational methods such as mental computation or estimation. Exercises include typical one- and two-step application problems, nonroutine problems and consumer problems. Nonroutine problems are exercises not normally taught or encountered in the curriculum, but understandable to the age group. Consumer problems deal primarily with the uses of mathematics in commercial situations (for example, the mathematics needed for buying and selling, including loans, percent, discount, finance

charge and reading advertisements) and are emphasized more at the 17-year-old level than at the two younger age levels.

An important new assessment category within numbers and numeration is estimation. There is an increasing need for students to be able to make good estimates. With the widespread use of the calculator, students need to be able to check the reasonableness of their answer by estimating. As consumers, they face daily decisions requiring estimation skills. Two types of estimation skills are assessed: computational estimation (which involves working with numerical data alone) and application estimation (which requires working with numerical data embedded in a real-world context).

To encourage students to estimate rather than use paper and pencil, only a few seconds are allowed to complete each exercise. Additional information on the assessment of estimation is included in Appendix B.

Variables and Relationships

The use of variables and relationships corresponds to an important part of the school mathematics curriculum. The exercises assessing skills in this area deal with the recognition of facts, definitions and symbols of algebra; the solution of equations and inequalities; the use of variables to represent problem situations and elements of a number system; the evaluation and interpretation of functions and formulas; the graphing of points and lines in a coordinate system; the use of exponential and trigonometric functions; and logic. There are very few exercises appropriate for 9-year-olds in this category, and only a few topics are appropriate for 13-year-olds. However, most exercises are appropriate at the 17-year-old level, where students have had the opportunity to study algebra.

Shape, Size and Position

The exercises in this content category measure objectives related to school geometry. The emphasis in the assessment is not on geometry as a formal, deductive system. Rather, the exercises concern plane and solid shapes, congruence, similarity, properties of triangles, properties of quadrilaterals, constructions, sections of solids, basic theorems and relationships, and rotations and symmetry.

Measurement

The measurement exercises cover appropriate units; equivalence relations; instrument reading; length, weight, capacity, time and temperature, perimeter, area and volume; non-

standard units; and precision and interpolation. A substantial number of the measurement exercises require the use and understanding of metric units.

Probability and Statistics

This content area reflects a greater emphasis on statistics and probability in the school mathematics curriculum. The exercises assess collecting data; organizing data with tables, charts, graphs; interpreting and analyzing data; drawing inferences; making generalizations; using basic statistics; predicting outcomes and determining combinations.

Technology

The impact of new technology on school mathematics is measured in this content area by assessing the use of the calculator and computer literacy.

Calculator

The increasing availability and popularity of calculators has made it important for National Assessment to gather information on their use by students. This was begun in the 1977-78 mathematics assessment.

Five categories of exercises are identified for assessment. They are: (1) routine computation, (2) more difficult computations, (3) understanding concepts, (4) exploration and (5) application or problem solving. Some calculator activities such as understanding and exploration are more appropriate for instructional use in the classroom and are not emphasized in the assessment. Thus, of the five categories of exercises, computation, nonroutine computation and application are measured in the greatest depth. Additional information on these categories can be found in Appendix B.

Computer Literacy

An increasing number of schools have computers or computer terminals available for students' use. This led National Assessment to begin collecting computer literacy data in the 1977-78 mathematics assessment.

Thirteen- and 17-year-olds are asked to provide background information on their experiences with computers: whether they have access to one in their school, what programming language and computer topics they have studied and what specific activities and problems

they have solved using a computer. The assessment of computer literacy includes measures of students' attitudes toward the uses, effect and role of computers and their knowledge of specific terms, flow charts or BASIC programs to determine the output.

A summary outline of these content categories is provided in Appendix C. The desired approximate percentages of exercises by content category and age group are shown in Figure 3.

FIGURE 3. Percentages of Exercises by Age and Content*

		Age 9	Age 13	Age 17
A.	Numbers and numeration	40%	40%	35%
B.	Variables and relationships	10	10	20
C.	Shape, size and position	10	10	10
D.	Measurement	15	15	10
E.	Probability and statistics	5	5	10
F.	Technology	10	10	10

**These percentages do not add to 100% because the attitudinal exercises are not included.*

Process

The process domain for the third assessment has five categories, as shown in Figure 4.

Like the content domain, the process domain can be used to classify either objectives of mathematics instruction or exercises to assess the learning of mathematics. Although each category suggests a type of mental process, neither objectives nor exercises falls neatly into a single process category — if only because the process has to be inferred; and different students may use different processes or different combinations of processes. Arbitrary decisions must be made in using any system of process categories. Such a system is helpful,

however, in ensuring consideration of the diversity possible within a given content category.

Mathematical Knowledge

Mathematical knowledge refers to the recall and recognition of mathematical ideas expressed in words, symbols or figures. Mathematical knowledge relies, for the most part, on memory processes. It does not ordinarily require any other more complex mental processes.

FIGURE 4. Percentages of Exercises by Age and Process Level

		Age 9	Age 13	Age 17
I.	Mathematical knowledge	20%	15%	15%
II.	Mathematical skill	25	25	25
III.	Mathematical understanding	20	25	25
IV.	Mathematical application	25	30	30
V.	Attitudes toward mathematics	10	5	5

Exercises that assess mathematical knowledge require that a student recall or recognize one or more items of information. An example of an exercise involving recall would be one that asks for a multiplication fact such as the product of five and two. Another example would be an exercise asking for the statement of a mathematical relationship such as the law of cosines. An example of an exercise involving recognition would be one that presents several symbols and asks which symbol means "parallel."

Mathematical Skill

Mathematical skill refers to the routine manipulation of mathematical ideas and relies on algorithmic processes that are standard procedures leading to answers. Exercises assessing mathematical skill assume that the required algorithm has been learned and practiced. They do not require that the student decide which algorithm to use or that he or she apply

the algorithm to a new situation. Such exercises aim at measuring proficiency in carrying out the algorithm rather than understanding how or why it works. Mathematical skill is assessed by exercises that require the performance of specified tasks, such as making measurements, multiplying two fractions, performing mental computations, graphing a linear equation or reading a table.

Mathematical Understanding

Mathematical understanding refers to the explanation and interpretation of mathematical knowledge and relies primarily on translation processes. The mathematical knowledge can be expressed in words, symbols or figures; and the translation may be within or between any of these modes of expression. Mathematical understanding involves memory processes of associating one item of knowledge with another.

Mathematical understanding may also require judgment in selecting the appropriate uses of different tools or processes. For example, students should understand appropriate times to use a calculator, computer, estimation or paper-and-pencil computation.

Exercises that assess mathematical understanding require that a student provide an explanation, an illustration for one or more items of knowledge or the transformation of knowledge. They do not require the application of that knowledge to the solution of a problem. An example of an exercise involving explanation is one that asks why a certain graph is not the graph of the function. Exercises involving transformation might ask for a drawing of an array to represent six times seven or ask for an equation to represent the information in a word problem.

Mathematical Application and Problem Solving

Mathematical application and problem solving refer to the *use* of mathematical knowledge, skill and understanding in solving both routine and nonroutine problems. Mathematical application and problem solving rely on memory and algorithmic, translative and judgmental processes. The student is not told how to solve the problem; reasoning and decision-making processes must be used.

Exercises that assess mathematical application and problem solving require a sequence of processes that relate to the formulation, solution and interpretation of problems. The processes may include recalling and recording knowledge, selecting and carrying out algorithms, making and testing conjectures, and evaluating arguments and results.

Exercises assessing mathematical application may vary from routine textbook problems to

exercises dealing with mathematical arguments. An exercise might require the solution of a standard problem or proportion, the demonstration that two geometric figures are congruent, an estimate of the amount of carpet needed for a room, or the formulation of a problem, given a graph of statistical data.

Exercises assessing problem solving require the use of strategies in solving nonroutine problems. Students may need to be able to use such strategies as drawing diagrams, trial and error, modeling, simplification and estimation in order to correctly solve the problems.

Attitudes Toward Mathematics

National Assessment assessed attitudes toward mathematics for the first time during the second assessment in 1977-78, and these exercises are reassessed in the third mathematics assessment. Five categories of attitudinal measures were developed: (1) mathematics in school, (2) mathematics and oneself, (3) mathematics and society, (4) mathematics as a discipline and (5) attitudes toward computers. These categories were not developed as scales but rather as sets of exercises with related content. Additional information on the assessment of attitudes appears in Appendix B.

The desired approximate percentages of exercises by age group and process category are shown in Figure 4.

Questions To Be Answered by the Assessment

In the development and selection of exercises for the assessment, care was taken to assure an appropriate balance of emphasis on both the content and process dimensions. Achievement of this balance was facilitated by a set of questions organized according to the categories of the process dimension. The questions were based on the combined priorities of the interested public, mathematicians, mathematics educators (including teachers) and educational administrators.

I. Mathematical knowledge

   A. How well can students recall and recognize facts, definitions and symbols?

II. Mathematical skill

   A. How well can students perform paper-and-pencil computations, including

computations with whole numbers, integers, fractions, decimals, percents, and ratios and proportions?

B. How well can students perform algebraic manipulations?

C. How well can students perform geometric manipulations like constructions and spatial visualizations?

D. How well can students make measurements?

E. How well can students read graphs and tables?

F. How well can students compute statistics, probabilities or combinations?

G. How well can students perform mental computations, including computation with whole numbers, fractions, decimals and percents?

H. How well can students estimate the answers to computations and measurements?

I. How well can students perform computations involving whole numbers, decimals, fractions and percents using calculators?

J. How well can students read flow charts or basic computer programs?

III. Mathematical understanding

A. How well can students translate a verbal statement into symbols or a figure, and vice versa?

B. How well do students understand mathematical concepts and principles?

C. How well can students select the appropriate uses of computers?

D. How well can students select an appropriate computational method such as paper and pencil, mental, estimation or calculator?

IV. Mathematical application

A. How well can students solve routine textbook problems?

B. How well can students solve nonroutine problems?

C. How well can students apply problem-solving strategies?

D. How well can students estimate the answers to application problems?

E. How well can students interpret data and draw conclusions?

F. How well can students use mathematics, including logic, in reasoning and making judgments?

G. How well can students use a calculator to solve application problems?

V. Attitudes

A. How do students feel about the mathematics they encounter in school?

B. How do students feel about the various activities in mathematics classes?

C. How do students feel about their personal experience with mathematics?

D. What are students' beliefs about the nature of mathematics as a discipline?

E. What are students' beliefs about the value of mathematics to society?

F. What are students' beliefs about computers?

The development and selection of exercises is primarily organized around these questions, which express the main objectives of the third mathematics assessment and will serve to organize the reports of assessment results.

To answer these questions as comprehensively as possible, sets of related or "nested" exercises appear in the assessment. For example, the same numbers may be used in a computational exercise and in an application exercise, or identical data may be provided in several different formats, or an intermediate step in a multistep problem may be assessed separately in another exercise. Nested exercises are an attempt to identify the mathematical processes that cause students difficulty.

APPENDIX A

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APPENDIX B

DETAILS OF OBJECTIVES FOR THE THIRD MATHEMATICS ASSESSMENT

For the interested reader, more information is provided here on certain assessment topics in the third assessment. Topics include assessment of attitudes, the calculator and estimation skills.

Attitudes

The attitudinal measures included in the third assessment were originally developed for the second assessment and will be reassessed to provide information on changes in students' attitudes. Five categories of attitudinal measures were developed.

The first category, mathematics in school, consists of exercises assessing attitudes toward the mathematics courses students have encountered in school. Subcategories of exercises include a school subject comparison, questions about the frequency of various classroom activities and students' attitudes toward those activities, and a breakdown of mathematics content activities.

The subject-comparison exercises list the most commonly encountered school subjects (e.g., science, mathematics, social studies) and ask respondents to rate each subject on the basis of whether they like or dislike it, whether they find it easy or hard and whether or not they view it as important.

Students are presented with a list of activities that might occur in a mathematics classroom and are asked to indicate how often (often, sometimes, never) they have participated in such activities. They are also asked to state whether they like or dislike each activity and whether they find the activity useful in helping them learn mathematics. Typical exercises are the following:

Last year in your mathematics course how often did you

A. watch the teacher work mathematics problems on the board?

Often	Sometimes	Never
⬭	⬭	⬭

B. get individual help from the teacher on your mathematics?

Often	Sometimes	Never
⬭	⬭	⬭

C. help a classmate do mathematics?

Often	Sometimes	Never
⬭	⬭	⬭

How do you feel about each of these activities in learning mathematics? First, how much do you like or dislike them? Second, how useful are they in learning mathematics? Indicate your feeling by filling in one oval on each line.

A. Taking mathematics tests

Like A Lot	Like	Undecided	Dislike	Dislike A Lot
⬭	⬭	⬭	⬭	⬭

Very Useful	Useful	Undecided	Not Very Useful	Useless
⬭	⬭	⬭	⬭	⬭

B. Doing mathematics homework

Like A Lot	Like	Undecided	Dislike	Dislike A Lot
⬭	⬭	⬭	⬭	⬭

Very Useful	Useful	Undecided	Not Very Useful	Useless
⬭	⬭	⬭	⬭	⬭

Respondents are also given a list of content activities and asked to respond on an important/not-important, easy/hard, like/dislike basis. A sample exercise is:

> How do you feel about each of these mathematics activities? First, how important are they? Second, how easy are they? Third, how much do you like them? Indicate your feeling by filling in one oval on each line.

A. Solving word problems

Very Important	Important	Undecided	Not Very Important	Not Important At All
Very Easy	Easy	Undecided	Hard	Very Hard
Like It A Lot	Like It	Undecided	Dislike It	Dislike It A Lot

B. Working with fractions

Very Important	Important	Undecided	Not Very Important	Not Important At All
Very Easy	Easy	Undecided	Hard	Very Hard
Like It A Lot	Like It	Undecided	Dislike It	Dislike It A Lot

C. Estimating answers to problems

Very Important	Important	Undecided	Not Very Important	Not Important At All
Very Easy	Easy	Undecided	Hard	Very Hard
Like It A Lot	Like It	Undecided	Dislike It	Dislike It A Lot

Exercises within the "mathematics and oneself" category assess a respondent's perception of himself or herself in relation to mathematics. Different attitudinal components such as anxiety, motivation, self-concept and enjoyment of mathematics are reflected in these Likert-type exercises. A sample exercise follows:

This exercise asks how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "True about me," "Sometimes true about me" and "Not true about me." For each part, choose the one response that best describes how you feel about the statement. Be sure to fill in one oval in each box.

A.	Mathematics is boring for me.		
	True About Me ⬭	Sometimes True About Me ⬭	Not True About Me ⬭
B.	I usually understand what we are talking about in mathematics.		
	True About Me ⬭	Sometimes True About Me ⬭	Not True About Me ⬭
C.	Doing mathematics makes me nervous.		
	True About Me ⬭	Sometimes True About Me ⬭	Not True About Me ⬭

The four examples above are for 9-year-olds. For 13- and 17-year-olds, the exercises have five response options, from "Strongly Disagree" to "Strongly Agree."

The "mathematics and society" category reflects two major concerns: the value of mathematics to the individual as a member of society and the value of mathematics to society in general. Likert-type exercises are designed to assess attitudes toward both the usefulness and importance of mathematics to society. A typical exercise follows:

This exercise asks how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "Strongly Disagree," "Disagree," "Undecided," "Agree" or "Strongly Agree." For each part, choose the one response that best describes how you feel about the statement. Be sure to fill in one oval in each box.

A. Most people do not use mathematics in their jobs.

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
○	○	○	○	○

B. I would like to work at a job that lets me use mathematics.

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
○	○	○	○	○

Attitudes toward mathematics as a field of study are the focus of the exercises in the "mathematics as a discipline" category. Students' views toward mathematics as a cumulative or compartmentalized subject, the status of mathematics as a fixed or changing subject and mathematics as a process are among the different aspects of attitude assessed in this category. Items such as the following are included:

This exercise asks how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "Strongly Disagree," "Disagree," "Undecided," "Agree" or "Strongly Agree." For each part, choose the one response that best describes how you feel about the statement. Be sure to fill in one oval in each box.

A. Mathematics is made up of unrelated topics.

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
○	○	○	○	○

B. Mathematics helps one to think logically.

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
○	○	○	○	○

C. Doing mathematics requires lots of practice in following rules.

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
○	○	○	○	○

Calculator

Like the assessment of attitudes, assessment of the use of the calculator was first done during the second mathematics assessment. At that time the increasing availability and popularity of calculators made it important for National Assessment to collect information on their use by students. The small, inexpensive calculator has the potential to dramatically influence the mathematics curriculum in the United States.

Several conferences were organized by National Assessment to discuss the place of the calculator in the second mathematics assessment. The participants at these conferences identified five categories of exercises for which calculators might be used. These categories are:

1. *Routine Computation* — This category includes typical computations with whole numbers, decimals, fractions and integers that are routinely taught at a particular age.

2. *More Difficult Computations* — Students might be asked to perform difficult computations or computations for which algorithms have not been formally taught. For example, 9-year-olds might be asked to do computations with decimals or difficult division problems. Thirteen-year-olds might be given chaining operations or conversions between fractions and decimals. Students at all ages might be required to work with very large numbers or complicated decimals that would make computation without a calculator tedious.

3. *Understanding Concepts* — Nine-year-olds might use the calculator to learn more about place value, and 13-year-olds might learn estimation for order of magnitude. Seventeen-year-olds might use the calculator to facilitate learning order of operations, approximating square roots, graphing functions and understanding properties of functions.

4. *Exploration* — Exercises in this category might deal with topics in number theory including series, summations, patterns or divisibility problems.

5. *Applications and Problem Solving* — This category includes routine and more difficult word problems and also multistep problems. For 9-year-olds, some problems might use larger numbers than they are accustomed to working with. Thirteen-year-olds might work problems involving percent, unit pricing and other more difficult word problems. Seventeen-year-olds might be given a variety of realistic consumer problems as well as exercises dealing with mathematical formulas.

In an effort to collect data on the use of the calculator, as wide a variety of topics as possible was assessed. Although it was agreed that the five categories should be assessed at each

of the three age groups, practical considerations imposed limitations on what could be assessed. Some activities using the calculator (especially understanding and exploration exercises) are more appropriate for instructional use in the classroom than for an assessment. Thus, of the five categories of exercises, categories 1, 2 and 5 were measured in the greatest depth.

One test booklet comprising approximately 20 exercises designed for calculator usage was administered at each age. Students were required to use a simple four-function calculator after being given a minimal amount of instruction on how the calculator is used. These exercises are difficult or too time-consuming without a calculator. Many of the exercises in each calculator test booklet appeared again in another booklet for the same age group. This allowed comparisons of performance with and without the calculator.

Each student answered background questions regarding his or her experience with calculators. These background questions asked how often the student has used a calculator, if the student's family owns one, in what school classes the student has used a calculator and what experiences with calculators outside of school the student has had.

Estimation

Estimation has long been recognized as an important mathematical skill. The widespread use and availability of calculators places additional importance on computational estimation skills, because it is easy to make a key-stroking error when using a calculator, students need to be able to estimate the magnitude of an answer or recognize the reasonableness of results.

Two types of estimation skills are assessed: computation estimation (which involves working with numerical data alone) and application estimation (which requires working with numerical data embedded in a real-world context). The prerequisite skill of mental computation of exact answers is also assessed.

Most estimation exercises included in the assessment require estimates using whole numbers, but some exercises include decimals and fractions. The four operations (addition, subtraction, multiplication and division) are represented, and several problems require knowledge of percent.

Both open-ended and multiple-choice exercises are assessed. For open-ended exercises, scoring guides indicating acceptable response ranges have been constructed. Generally, three categories of responses are considered acceptable: (1) exact answers, (2) very close estimates and (3) reasonable estimates. The intervals for these guides were determined by analyzing field-test results.

Sample exercises for computation estimation are:

A. 48 X 25 X 12

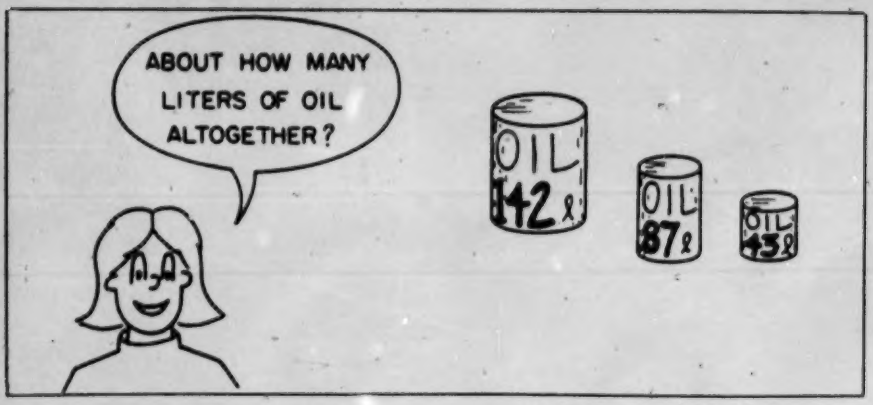
Estimate _____

B. 613 - 490

- 100
- 200
- 300
- 400
- 1,200
- I don't know.

Sample exercises for application estimation are:

Estimate _____



- 200
- 280
- 350
- 1,400
- I don't know.

An effort will be made to relate the arithmetic required in the mental computation exercises with the arithmetic needed in the computation and application estimation exercises. For example, a mental computation exercise might be 20 x 40. The related computation estimation exercise might be 19 x 42, which the student would probably solve by rounding each number to the nearest 10. A related application estimation exercise might ask students to estimate the area of a rectangle with a width of 19 and a length of 42. By embedding related or identical numbers in different types of problems, more information can be obtained on students' estimation skills.

APPENDIX C

CONTENT OUTLINE

A. Numbers and Numeration

1. Numeration (whole numbers, fractions, decimals, percent, integers, scientific notation)
2. Number concepts (whole numbers, fractions, decimals, percent, integers)
3. Operations (whole numbers, fractions, decimals, percents, integers)
4. Mental computation
5. Estimation
6. Properties
7. Relations

B. Variables and Relationships

1. Facts, definitions and symbols
2. Use of variables in equations and inequalities (solutions, equivalences and translations)
3. Operations with variables
4. Use of variables to represent elements of a number system
5. Functions and formulas
6. Coordinate systems

7. Exponential and trigonometric functions

8. Logic

C. Shape, Size and Position

1. Recognition of figures

2. Constructions and drawings

3. Visualization (static and dynamic)

4. Recognition of relationships (congruence, similarity and symmetry)

5. Identification of properties from given visual information within, between or among figures

6. Relationships involving classes of figures

7. Definitions, postulates and theorems (recall, inference and application)

D. Measurement

1. Unit (appropriate size and type of unit, unit equivalents, conversions within a system)

2. Instrument reading (English and metric rulers, scales, thermometers, clocks, etc.)

3. Linear measure (including nonstandard units)

4. Area, perimeter and volume

5. Precision

6. Estimation of measurements

E. Probability and Statistics

1. Organizing, displaying and interpreting information (tallies, graphs, charts and tables)

2. Measures of central tendency (mean, median, mode)

3. Measures of spread and position (range, percentile, standard deviation)

4. Sampling and polling

5. Probability (simple, compound and independent events; odds)

6. Combinations and permutations

F. Technology

1. Hand calculator

2. Computer literacy

LIST OF REFERENCES

Anick, Constance M. et al. "Minorities and Mathematics: Results From National Assessment," *The Mathematics Teacher,* vol. 74, 1981.

Bestgen, B.J. "Making and Interpreting Graphs and Tables: Results and Implications From National Assessment," *The Arithmetic Teacher,* vol. 28, 1980.

Carpenter, T.P. et al. "A Perspective of Students' Mastery of Basic Skills," in *Selected Issues in Mathematics Education,* ed. Mary Montgomery Lindquist. Chicago, Ill.: National Society for the Study of Education; Reston, Va.: National Council of Teachers of Mathematics, 1981.

Carpenter, T.P. et al. "Calculators in Testing Situations: Results and Implications From National Assessment," *The Arithmetic Teacher,* vol. 28, 1981.

Carpenter, T.P. et al. "Decimals: Results and Implications of the Second NAEP Mathematics Assessment," *The Arithmetic Teacher,* vol. 28, 1981.

Carpenter, T.P. et al. "NAEP Note: Problem Solving," *The Mathematics Teacher,* vol. 73, 1980.

Carpenter, T.P. et al. "National Assessment: Implications for the Curriculum of the 1980's," in *Research in Mathematics Education: Implications for the 80s,* ed. Elizabeth Fennema. Washington, D.C.: Association for Supervision and Curriculum Development (forthcoming).

Carpenter, T.P. et al. "Notes From NAEP: Perimeter and Area," *The Arithmetic Teacher,* vol. 22, 1975.

Carpenter, T.P. et al. "Notes From National Assessment: Addition and Multiplication With Fractions," *The Arithmetic Teacher,* vol. 22, 1975.

Carpenter, T.P. et al. "Notes From National Assessment: Basic Concepts of Area and Volume," *The Arithmetic Teacher,* vol. 22, 1975.

Carpenter, T.P. et al. "Notes From National Assessment: Estimation," *The Arithmetic Teacher,* vol. 23, 1976.

Carpenter, T.P. et al. "Notes From National Assessment: Processes Used on Computational Exercises," *The Arithmetic Teacher,* vol. 23, 1976.

Carpenter, T.P. et al. "Notes From National Assessment: Recognizing and Naming Solids," *The Arithmetic Teacher,* vol. 23, 1976.

Carpenter, T.P. et al. "Notes From National Assessment: Word Problems," *The Arithmetic Teacher,* vol. 23, 1976.

Carpenter, T.P. et al. "Problem Solving in Mathematics: National Assessment Results," *Educational Leadership,* vol. 37, 1980.

Carpenter, T.P. et al. "Research Implications and Questions From the Year 04 NAEP Mathematics Assessment," *Journal for Research in Mathematics Education,* vol. 7, 1976.

Carpenter, T.P. et al. "Results and Implications of the NAEP Mathematics Assessment: Elementary School," *The Arithmetic Teacher,* vol. 22, 1975.

Carpenter, T.P. et al. "Results and Implications of the NAEP Mathematics Assessment: Secondary School," *The Mathematics Teacher,* vol. 68, 1975.

Carpenter, T.P. et al. "Results and Implications of the Second NAEP Mathematics Assessment: Elementary School," *The Arithmetic Teacher,* vol. 27, 1980.

Carpenter, T.P. et al. *Results From the Second Assessment of the National Assessment of Educational Progress,* ed. M.K. Corbitt. Reston, Va.: National Council of Teachers of Mathematics, 1981.

Carpenter, T.P. et al. "Results of the Second NAEP Mathematics Assessment," *The Mathematics Teacher,* vol. 73, 1980.

Carpenter, T.P. et al. "Solving Verbal Problems: Results and Implications From National Assessment," *The Arithmetic Teacher,* vol. 28, 1980.

Carpenter, T.P. et al. "Students' Affective Responses to Mathematics: Results and Implications From National Assessment," *The Arithmetic Teacher,* vol. 28, 1980.

Carpenter, T.P. et al. "Students' Affective Responses to Mathematics: Secondary School Results From National Assessment," *The Mathematics Teacher,* vol. 73, 1980.

Carpenter, T.P. et al. "Subtraction: What Do Students Know?" *The Arithmetic Teacher,* vol. 22, 1975.

Carpenter, T.P. et al. "The Current Status of Computer Literacy: NAEP Results for Secondary Students," *The Mathematics Teacher,* vol. 73, 1980.

Carpenter, T.P. et al. "What Are the Chances of Your Students Knowing Probability?" *The Mathematics Teacher,* vol. 74, 1981.

Changes in Mathematical Achievement, 1973-78, Report no. 09-MA-01, 1972-73 and 1977-78 Assessments. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1979. ERIC no. ED 177 011. ISBN 0-89398-134-6.

Consumer Math: Selected Results From the First National Assessment of Mathematics, Report no. 04-MA-02, 1972-73 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1975. ERIC no. ED 111 696. ISBN 0-89398-127-3.

Fennema, E. and T.P. Carpenter. "Sex Differences in Mathematics: Results From National Assessment," *The Mathematics Teacher,* vol. 74, 1981.

Hiebert, J. "Units of Measure: Results and Implications From the Second NAEP Mathematics Assessment," *The Arithmetic Teacher,* vol. 28, 1981.

Hirstein, J. "The Second National Assessment in Mathematics: Area and Volume," *The Mathematics Teacher,* vol. 74, 1981.

Kerr, Donald. "A Geometry Lesson From National Assessment," *The Mathematics Teacher,* vol. 74, 1981.

Math Fundamentals: Selected Results From the First National Assessment of Mathematics, Report no. 04-MA-01, 1972-73 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1975. ERIC no. ED 102 029. ISBN 0-89398-126-5.

Mathematical Applications, Report no. 09-MA-03, 1977-78 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1979. ERIC no. ED 176 965. ISBN 0-89398-136-2.

Mathematical Knowledge and Skills, Report no. 09-MA-02, 1977-78 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1979. ERIC no. ED 176 964. ISBN 0-89398-135-4.

Mathematical Understanding, Report no. 09-MA-04, 1977-78 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1979. ERIC no. ED 182 174. ISBN 0-89398-137-0.

Mathematics Objectives, no. 04-MA-10, 1972-73 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1970. ERIC no. ED 063 140. ISBN 0-89398-125-7.

Mathematics Objectives, Second Assessment, no. 09-MA-10, 1977-78 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1978. ERIC no. ED 156 439. ISBN 0-89398-133-8.

Mathematics Technical Report: Exercise Volume, Report no. 04-MA-20, 1972-73 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1977. ERIC no. ED 138 468. ISBN 0-89398-129-X.

Mathematics Technical Report: Summary Volume, Report no. 04-MA-21, 1972-73 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1976. ERIC no. ED 129 636. ISBN 0-89398-130-3.

Mathematics Technical Report: Summary Volume, Report no. 09-MA-21, 1977-78 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1980. ERIC no. ED 186 279. ISBN 0-89398-142-7.

McKillip, W.D. "Computational Skill in Division: Results and Implications of the Second NAEP Mathematics Assessment," *The Arithmetic Teacher,* vol. 28, 1981.

Post, Thomas R. "Fractions: Results and Implications From National Assessment," *The Arithmetic Teacher,* vol. 28, 1981.

Procedural Handbook: 1977-78 Mathematics Assessment, Report no. 09-MA-40. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1980. ERIC no. ED 186 280. ISBN 0-89398-143-5.

Rathmell, Edward C. "Concepts of the Fundamental Operations: Results and Implications From National Assessment," *The Arithmetic Teacher,* vol. 28, 1980.

Results From the First Mathematics Assessment of the National Assessment of Educational Progress. Reston, Va.: National Council of Teachers of Mathematics, 1978.

The First National Assessment of Mathematics: An Overview, Report no. 04-MA-00, 1972-73 Assessment. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1975. ERIC no. ED 127 198. ISBN 0-89398-128-1.

The Second Assessment of Mathematics, 1977-78, Released Exercise Set, no. 09-MA-25. Denver, Colo.: National Assessment of Educational Progress, Education Commission of the States, 1979. ERIC no. ED 187 543. ISBN 0-89398-138-9.